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Computational morphogenesis of free form shell structures by optimization

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Abstract

Shell structures in civil engineering are known extremely parameter sensitive, so the aim of computational morphogenesis of free-form shells is to find a rational shape which defines an ideal internal stress state. This paper sets forth a kind of computational method to generate free-form shell structures, which is a synthesis of design modelling, structural analysis and mathematical optimization. At first the design modelling of the shell is completed by non-uniform rational b spline (NURBS), then structural analysis is executed to the initial shape, at last the NLPQL algorithm which is a SQP algorithm is used for the optimization. During the optimization metamodeling is used, which involves sensitivity analysis, design of experiments and regression analysis. Several examples show the power of the approach.

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Keywords: shell; computational morphogenesis; NURBS; metamodeling; NLPQL.

1. Introduction

Free Form Design and Structural Morphology that are two groups of the International Association for Shell and Spatial Structures (IASS) focus on the shape design and the interaction between the shape and the structure behavior. To find the ideal shape, the initial methodology within the form-finding field (or

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morphogenesis as is modernly named) is the use of physical models. This design approach is quite old, and some pioneers were C. Wren, G. Poleni, J.B. Rondelet and A. Gaudí[1]. As for the new form-finding concept of the free forms, i.e. forms in which the conception and generation are independent of the geometry, Heinz Isler [2-3] is an unquestionable precursor in the design and construction.

Diverse design techniques based on numerical methods have been developed allowing the form-finding so far, which is called computational morphogenesis. Computational morphogenesis is realized on the firm foundation of both FEM(finite element method) as a tool of numerical analysis and various kinds of method based on relatively newly developed algorithms for structural optimization [4]. Form finding of free form shell structures by traditional sensitivity-based shape optimization has been done by many researchers like Ramm and Mehlhorn [5], Bletzinger [6-7], Espath L. F. R.[8]. Besides form finding by topological optimization has also been done by some authors like Maurin and Motro [9], Cui C[10] Yang X Y [11]. I Vizotto[12-13] adopts finite element method for form finding. Response surface method (RSM) has been successfully used in optimization [14-15].

In this paper, computational morphogenesis of free form shell structures by shape optimization is presented using RSM with the geometry modeled by NURBS.

2. Theoretical aspects

The computational morphogenesis of free form surface structures by optimization in this paper uses metamodel, which is synonymous with response surface method (RSM). In the case of shape optimization in civil engineering, once the response surface of a structure has been constructed, optimization is reduced to the task of finding the smallest value on the response surface. The main steps include the following.

- Describing the free form surface by non-uniform rational b-spline (NURBS)
- The selection of the structural parameters and the definition of a number of “level” for each selected parameters, by using the design of experiments (DOE) techniques.
- In design space, the response features are calculated by carrying out finite element analysis (FEA).
- Performing the regression to create the response surface model of the structure.
- Doing the optimization with non-linear programming by quadratic lagrangian (NLPQL) on the response surface, by which the relation between the response features and structure parameters is expressed explicitly.

2.1. Geometry modeling by NURBS

In order to represent a free form surface, a parametric representation is used. NURBS parameterization is well suitable for shape optimization in any physical problem involving curves, surfaces and solids. For shell structures, surface is represented by two dimensional NURBS. The surface is completely defined by a net of control points, a vector of weights and the polynomial functions degree in the parametrical directions. One of the most important features of the NURBS is the local control: with respect to the Bezier curves, the presence of a weight associated to each control point allows to locally modify the surface by changing the control point coordinate, without modify the rest of the surface. This property is important when the position of some points is fixed, as in the case of constrained points, or when one or more edges of the surface is defined. Section headings

2.2. Sampling and parameter selection

To create a response surface that will serve as a surrogate for the FE simulation model, the basic process is one of calculating predicted values of the response features at sample points in the parameter space by performing an experiment at each of those points. A number of feature values from the experiment run across the parameter domain are fit with a response surface.

The term experiment herein refers to either physical experiments or computer experiments. The planning of experimentation is further referred to design of experiments (DOE). In the current study, the CCD method in DOE is used in the paper as it is simple in constructing the response surfaces of a polynomial type. [16]. The parameter selection requires a considerable physical insight into the target structure, and many methods can be used such as by experience or by performing a parameter effect analysis (hypothesis testing) based on statistical variance (square of the standard deviation) analysis. During the process of computational morphogenesis, the parameters besides their domain which can influence the architectural esthetics that usually is considered by the architects and the structural performance that is usually valued by structure engineers are selected. In this light, the interaction between the architects and structure engineers are fully reflected.

2.3. Finite element analysis

After the sample points are gained by DOE, the response features are calculated by carrying out finite element analysis. The surface needs to be discretized into a suitable structural mesh and a linear or non linear structural finite element analysis can be performed, in order to evaluate the displacements, the stress, the total strain energy, the buckling load or the dynamic behavior.

2.4. Response surface regression

The selected response surface form should be capable of attaining surfaces that meet specific smoothness requirements of an application. There is often a balance between assumptions and data requirement. Polynomials are used to represent a response surface because the calculations are simple and the result. In this paper, a quadratic polynomial response surface is used, and the form is:

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \sum_{j=1}^k \beta_{ij} x_i x_j \quad (1)$$

where $\beta_0, \beta_i, \beta_{ij}$ are the regression coefficients to be estimated from the experimental data. The method of least-square fitting is usually used in the coefficient estimation process to create a response surface. Before the regressed response surface is put forward to be used in optimization, it should be verified to check whether the regressed surface has enough accuracy. R^2 (ranged from 0.0 to 1.0) criterion, empirical integrated squared Error (EISE) criterion and the root mean squared error (RMSE) can be used in response surface verification.

2.5. Shape optimization by NLPQL

the regressed response surface After the regression, the optimization using the response surface function can be carried out by which new free form shell with good structural performance or less cost would be obtained. The mathematic optimization method used in this paper is NLPQL which is a kind of SQP [17].

In this paper, the total strain energy is chosen as the optimized objective, by minimizing which the shell structural behaviour can be improved substantially.

3. Applications and results

3.1. Parabolic roof shell

The case of shape optimization of a concrete shell used by Bletzinger and Ramm [7] is studied in sensitivity-based method. It is subjected to its own weight and a vertical uniform load, for different design criteria. In this paper a parabolic roof shell is optimized by RSM. In fig. 1(a), the shell of rectangular plan ($b = 6\text{m}$, $l = 12\text{m}$) and uniform constant thickness ($t = 0.05\text{m}$) is supported by diaphragms at the smaller edges, both coordinates of $s1$ and $s2$ are set to 3m . The structure is also subjected to its own weight and a vertical uniform vertical load $p = 5\text{ kN/m}^2$. Support conditions are fixed hinges. The design variables are $s1$ and $s2$, and the domain of these two variables are $3\text{m} < s1 (s2) < 6\text{m}$. The material properties are $E = 3 \times 10^4\text{Mpa}$, $\nu = 0.2$. During the FEA, large deflection is considered. The total strain energy is chosen as the objective function with no design constraint. After the optimization by RSM, the total strain energy decreases to $5.73\text{e}5\text{J}$ from $3.99\text{e}6\text{J}$, and the resulting shape is fig. 1(b) with $s1 = 5.881\text{m}$, $s2 = 3.045\text{m}$.

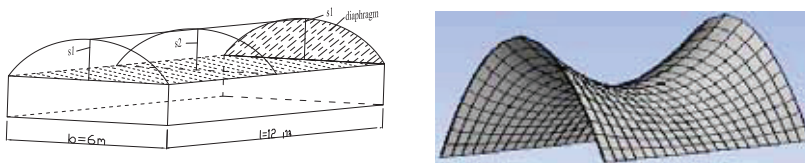


Fig.1. (a) the initial parabolic roof shell; (b) the resulting shape

3.2. One protruded and concaved free form surface

One protruded and concaved free form surface is created by NURBS, see Fig.2 (a). There vertical positions of some points are assumed as the design variables with $r1 = 3\text{m}$, $r2 = 2\text{m}$, $r3 = 3\text{m}$. The thickness is 15mm and the material properties of concrete are $E = 2.1 \times 10^5\text{Mpa}$, $\nu = 0.3$. The structure is loaded by a vertical uniform pressure 5 kN/m^2 with fixed supports at all edges. To obtain a stiff structure, minimizing the maximum vertical displacement of the structure, the total strain energy is chosen as the objective function without any constraint. The domain of variables is from 0m to 5m . After the optimization, the total strain energy that is $2.67\text{e}5\text{J}$ for the initial model decreases to $2.08\text{e}5\text{J}$, which means that the surface becomes stiffer. The shape after optimization is $r1 = 4.998\text{m}$, $r2 = 4.529\text{m}$, $r3 = 0.073\text{m}$, see Fig.2 (b).



Fig.2. (a) the initial protruded and concaved free form surface; (b) the protruded and concaved free form surface after optimization

4. Concluding remarks

This paper presents the method of structural optimization as general computational tools to find the shape subjected to different load cases and certain boundary conditions. A quadratic polynomial response surface is constructed using the finite element model simulation data and employed as objective function or design constraint, which different from the traditional sensitivity-based optimization. Once the response surface is constructed, there is no finite element calculation involved in each optimization iteration. With NURBS and optimization by RSM, the interaction between the architects and the structure engineers can be easy and positive; two free form surface shells are optimized by this way.

A small number of design parameters chosen in the examples and an estimation of the response surfaces by a quadratic function may not be far from the truth for more general problems. It is still a challenge to test the response surface method for computational morphogenesis of more complex civil engineering structures where the relation between the design variables and the response quantities of interest is more complex

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